Some Polygonal Sum Labeling of Bistar

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ABSTRACT- A (p, q) graph G is said to admit a polygonal sum labeling if its vertices can be labeled by non -negative integers such that the induced edge labels obtained by the sum of the labels of end vertices are the first q polygonal numbers. A graph G which admits a polygonal sum labeling is called a polygonal sum graph. In this paper we have proved that the Bistar ($B_{n,n}$) admit Pentagonal, Hexagonal, Heptagonal, Octagonal, Nonagonal and Decagonal Sum Labeling.

Keywords: Polygonal sum labeling and polygonal sum graph.

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [2]. A graph labeling is an assignment of integers to the vertices or edges or both subjects to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [5]. In the recent years, dozens of graph labeling techniques have been studied in over 1500 papers [3]. Triangular sum labeling was discussed in [6]. In [4] some polygonal sum labeling of paths were discussed. In this paper we have proved that the graph $B_{n,n}$ admit Pentagonal, Hexagonal, Heptagonal, Octagonal, Nonagonal and Decagonal sum labeling.

2. DEFINITIONS

2.1 Definition

Polygonal numbers are just the number of vertices formed by a certain polygon. The first number in any group of polygonal numbers is always one or a point. The second number is equal to the number of vertices of the polygon. The third number is made by extending two of the sides of the polygon from the second polygonal number, completing the larger polygon and placing vertices and other points wherever necessary. The third polygonal number is found by adding all the vertices and points in the resulting.

2.2 Definition

Pentagonal numbers are numbers that create a pentagon. In other words 1, 5, 12, 22, 35, 51, 70, 92, 117, 145 . . . are pentagonal numbers.

The n^{th} pentagonal number is denoted by A_n, then $A_n = \frac{n(3n-1)}{2}$.

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2.3 Definition

A pentagonal sum labeling of a graph G is one to one function f: V(G)→ N that induces a bijection f^{+} : E(G) → $\{A_1, A_2, . . . , A_q\}$ of the edges of G defined by $f^{+}(uv)$ = f(u) + f(v) for every e = uv ϵ E(G). The graph which admits such labeling is called a pentagonal sum graph.

2.4 Definition

Hexagonal numbers are numbers that create a hexagon. In other words 1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276, . . . are hexagonal numbers.

The n^{th} hexagonal number is denoted by B_n, then $B_n = n(2n-1)$.

2.5 Definition

A hexagonal sum labeling of a graph G is a one-to-one function f : V(G) → N that induces a bijection f^{+} : E(G) → $\{B_1, B_2, ... B_q\}$ of the edges of G defined by $f^{+}(uv)$ = f(u) + f(v) for every e= uv ϵ E(G). The graph which admits such labeling is called a hexagonal sum graph.

2.6 Definition

Heptagonal numbers are numbers that create a heptagon. In other words 1, 7, 18, 34, 55, 81, 112, 148, 189, 235, 286, 342, 403, 469, 540, 616, 697, . . . are heptagonal numbers.

The n^{th} heptagonal number is denoted by C_n, then $C_n = \frac{n(5n-3)}{2}$.

2.7 Definition

A Heptagonal sum labeling of a graph G is a one-to-one function f : V(G) → N that induces a bijection f^{+} : E(G) → $\{C_1, C_2, . . . , C_q\}$ of the edges of G defined by $f^{+}(uv)$ = f(u) + f(v) for every e= uv ϵ E(G). The graph which admits such labeling is called a heptagonal sum graph.

2.8 Definition

Octagonal numbers are numbers that create an octagon. In other words 1, 8, 21, 40, 65, 96, 133,176,225,280,341,408,481,560, 645, 736, 833, 936, 1045, 1160, 1281, 1408, 1541, . . . are octagonal numbers.

The n^{th} octagonal number is denoted by D_n, then $D_n = n(3n-2)$.

2.9 Definition

An octagonal sum labeling of a graph G is a one-to-one

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function f : V(G) → N that induces a bijection f^+ : E(G) → $\{D_1, D_2, \ldots, D_q\}$ of the edges of G defined by $f^+(uv) = f(u) + f(v)$ for every e= uv ϵ E(G). The graph which admits such labeling is called an octagonal sum graph.

2.10 Definition

Nonagonal numbers are numbers that create a nonagon. In other words: 1, 9, 24,46, 75, 111, 154, 204, 261, 325, 396, 474, 559, 651, 750, 856, 969, 1089, 1216, 1350, 1491, 1639, 1794,1956, 2125, 2301, 2484, 2674, 2871, 3075, . . . are nonagonal numbers.

The n^th nonagonal number is denoted by E_n, then

$$E_n = \frac{n(7n-5)}{2}$$

2.11 Definition

A nonagonal sum labeling of a graph G is a one-to-one function f : V(G) → N that induces a bijection f^+ : E(G) → $\{E_1, E_2, \ldots, E_q\}$ of the edges of G defined by $f^+(uv) = f(u) + f(v)$ for every e= uv ϵ E(G). The graph which admits such labeling is called a nonagonal sum graph.

2.12 Definition

Decagonal numbers are numbers that create a decagon. In other words:1, 10, 27, 52, 85, 126, 175, 232, 297, 370, 451, 540, 637, 742, 855, 976, 1105, 1242, 1387, 1540, 1701, 1870, 2047, 2232, 2425, 2626, 2835, . . . are decagonal numbers.

The n^th decagonal number is denoted by F_n, then

$$F_n = n(4n-3).$$

2.13 Definition

A decagonal sum labeling of a graph G is a one-to-one function f : V(G) → N that induces a bijection f^+ : E(G) → $\{F_1, F_2, \ldots, F_q\}$ of the edges of G defined by $f^+(uv) = f(u) + f(v)$ for every e= uv ϵ E(G). The graph which admits such labeling is called a decagonal sum graph.

3. MAIN RESULTS

Here we have proved that the graph $B_{n,n}$ admit Pentagonal, Hexagonal, Heptagonal, Octagonal, Nonagonal and Decagonal Sum Labeling.

3.1 Theorem

The graph $B_{n,n}$ is a pentagonal sum graph for $n \geq 2$.

Proof:

Let $V(B_{n,n}) = \{u_i, 1 \leq i \leq n\} \cup \{v_i, 1 \leq i \leq n\} \cup \{u\} \cup \{v\}$

$E(B_{n,n}) = \{e_i, 1 \leq i \leq 2n+1\}$ (See Fig.1)

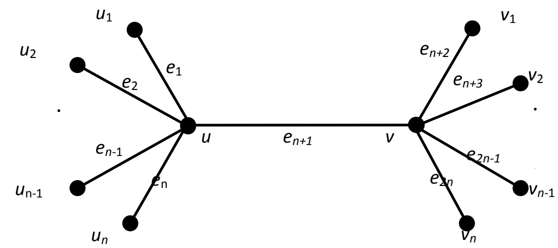

Fig.1: Ordinary labeling of $B_{n,n}$

First we label the vertices of $B_{n,n}$ as follows:

Define f: V $(B_{n,n})$ →N by

$$f(u) = 0,$$
$$f(u_i) = \frac{i(3i-1)}{2}, 1 \leq i \leq n$$
$$f(v) = \frac{(n+1)(3n+1)}{2},$$
$$f(v_i) = (3n+4)i + \frac{3i(i-1)}{2}, 1 \leq i \leq n$$

Then the induced edge labels are,

$$f(e_i) = \frac{i(3i-1)}{2}, 1 \leq i \leq 2n+1,$$

Therefore,

$$f^+(E(B_{n,n})) = \{1, 5, 12, 22, 35, 51, 70, 92, 117, \ldots\}$$

Therefore, f is a pentagonal sum labeling and hence $B_{n,n}$ is a pentagonal sum graph.

Pentagonal sum labeling of $B_{7,7}$ is shown in Fig

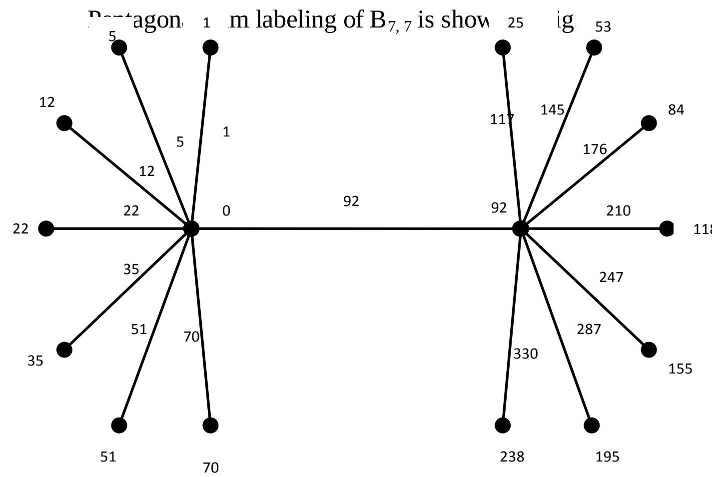

Fig.2: Pentagonal sum labeling of $B_{7,7}$

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3.2 Theorem

The graph $B_{n,n}$ is a hexagonal sum graph for $n \geq 2$.

Proof:

Let $V\left(B_{n,n}\right)=\{u_i, 1\leq i\leq n\}\cup\{v_i, 1\leq i\leq n\}\cup\{u\}\cup\{v\}$

$E\left(B_{n,n}\right)=\{e_i, 1\leq i\leq 2n+1\}$ (See Fig.1)

First we label the vertices of $B_{n,n}$ as follows:

Define f: V ($B_{n,n}$) →N by

$$f(u)=0,$$
$$f(u_i)=i(2i-1),\ 1\leq i\leq n$$
$$f(v)=(n+1)(2n+1),$$
$$f(v_i)=(4n+9)i-4,\ 1\leq i\leq n$$

Then the induced edge labels are,

$$f(e_i)=i(2i-1),\ 1\leq i\leq 2n+1,$$

Therefore,

$$f^{+}\left(E\left(B_{n,n}\right)\right)=\{1,\ 6,\ 15,\ 28,\ 45,\ 66,\ 91,\ 120,...\}$$

Therefore f is a hexagonal sum labeling and hence $B_{n,n}$ is a hexagonal sum graph.

Hexagonal labeling of $B_{4,4}$ is shown in Fig.3

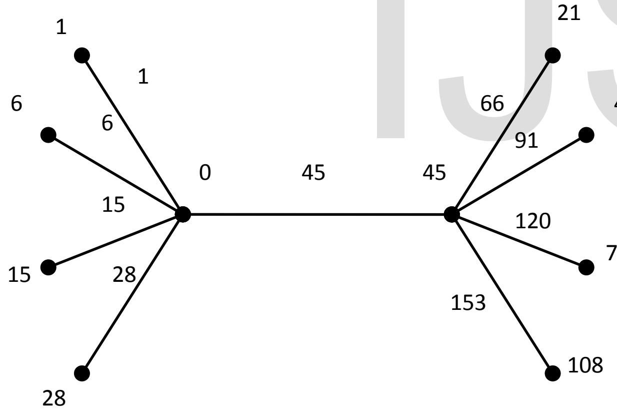

Fig.3: Hexagonal labeling of $B_{4,4}$

3.3 Theorem

The graph $B_{n,n}$ is a heptagonal sum graph for $n \geq 2$.

Proof:

Let $V\left(B_{n,n}\right)=\{u_i, 1\leq i\leq n\}\cup\{v_i, 1\leq i\leq n\}\cup\{u\}\cup\{v\}$

$E\left(B_{n,n}\right)=\{e_i, 1\leq i\leq 2n+1\}$ (See Fig.1)

First we label the vertices of $B_{n,n}$ as follows:

Define f: V ($B_{n,n}$) →N

$$f(u)=0,$$
$$f(u_i)=\frac{i(5i-3)}{2},\ 1\leq i\leq n$$
$$f(v)=\frac{(n+1)(5n+2)}{2},$$
$$f(v_i)=(5n+6)i+\frac{5i(i-1)}{2},\ 1\leq i\leq n$$

Then the induced edge labels are,

$$f(e_i)=\frac{i(3i-1)}{2},\ 1\leq i\leq 2n+1,$$

Therefore,

$$f^{+}\left(E\left(B_{n,n}\right)\right)=\{1,\ 7,\ 18,\ 34,\ 55,\ 81,\ 112,\ 148,\ 189,\ 235,...\}$$

Therefore, f is a heptagonal sum labeling and hence $B_{n,n}$ is a heptagonal sum graph.

Heptagonal labeling of $B_{10,10}$ is shown in Fig.4

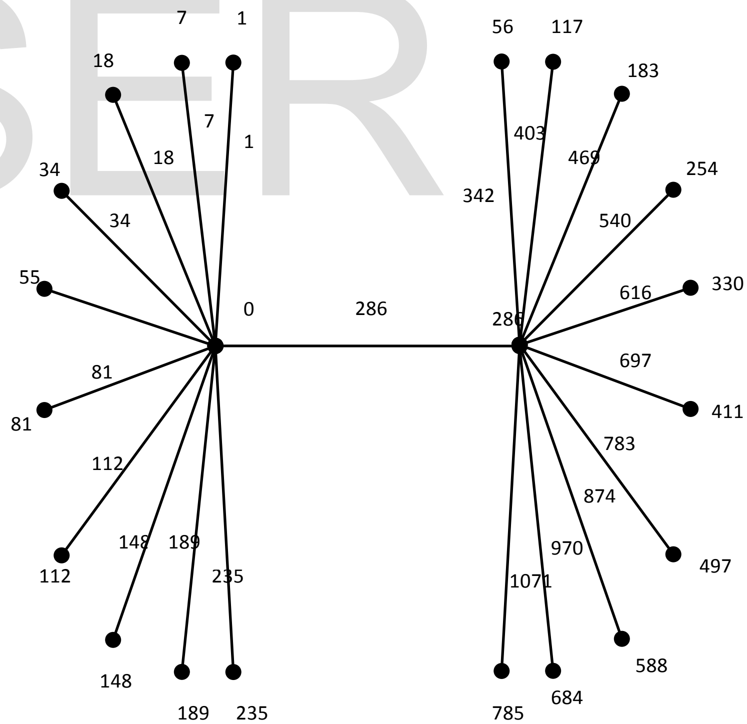

Fig.4: Heptagonal labeling of $B_{10,10}$

3.4 Theorem

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The graph $B_{n,n}$ is a octagonal sum graph for $n \geq 2$.

Proof:

Let $V\left(B_{n,n}\right)=\{u_i, 1\leq i\leq n\}\cup\{v_i, 1\leq i\leq n\}\cup\{u\}\cup\{v\}$

$E\left(B_{n,n}\right)=\{e_i, 1\leq i\leq 2n+1\}$ (See Figure 1)

First we label the vertices of $B_{n,n}$ as follows:

Define f: V ($B_{n,n}$) →N by

$$f(u)=0,$$
$$f(u_i)=i(3i-2), 1\leq i\leq n$$
$$f(v)=(n+1)(3n+1),$$
$$f(v_i)=(6n+7)i+3i(i-1), 1\leq i\leq n$$

Then the induced edge labels are,

$$f(e_i)=i(3i-2), 1\leq i\leq 2n+1,$$

Therefore,

$$f^{+}\left(E\left(B_{n,n}\right)\right)=\{1, 8, 21, 40, 65, 96, 133, 176, \ldots\}$$

Therefore, f is a octagonal sum labeling and hence $B_{n,n}$ is a octagonal sum graph.

Octagonal labeling of $B_{5,5}$ is shown in Fig.5

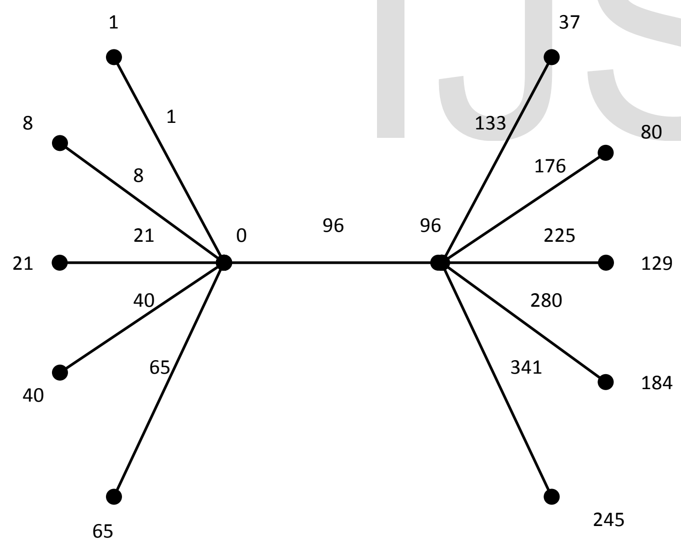


Fig.5: Octagonal labeling of $B_{5,5}$

3.5 Theorem

The graph $B_{n,n}$ is a nanogonal sum graph for $n \geq 2$.

Proof:

Let $V\left(B_{n,n}\right)=\{u_i, 1\leq i\leq n\}\cup\{v_i, 1\leq i\leq n\}\cup\{u\}\cup\{v\}$

$E\left(B_{n,n}\right)=\{e_i, 1\leq i\leq 2n+1\}$ (See Figure 1)

First we label the vertices of $B_{n,n}$ as follows:

Define f: V ($B_{n,n}$) →N by

$$f(u)=0,$$
$$f(u_i)=\frac{i(7i-5)}{2}, 1\leq i\leq n$$
$$f(v)=\frac{(n+1)(7n+2)}{2},$$
$$f(v_i)=(7n+8)i+\frac{7i(i-1)}{2}, 1\leq i\leq n$$

Then the induced edge labels are,

$$f(e_i)=\frac{i(7i-5)}{2}, 1\leq i\leq 2n+1,$$

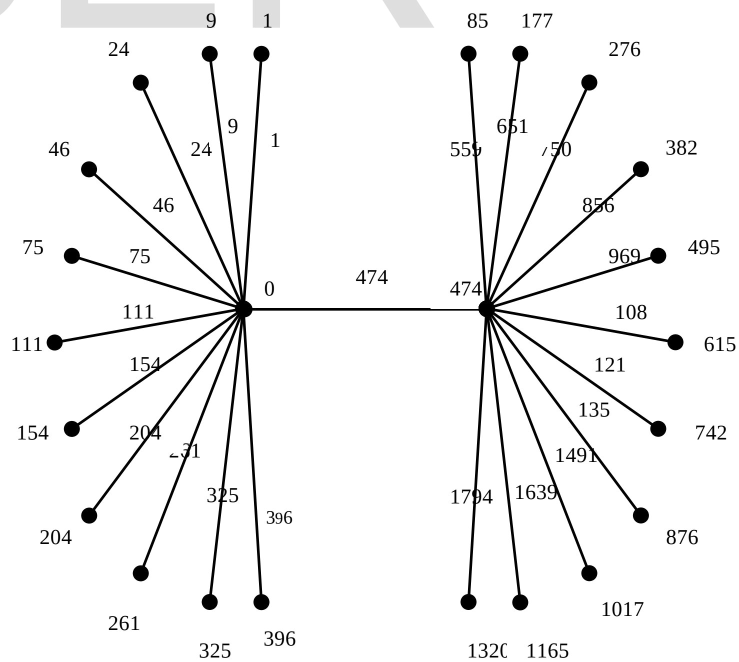
Therefore,

$$f^{+}\left(E\left(B_{n,n}\right)\right)=\{1, 9, 24, 46, 75, 111, 154, 204, 261, 325, 396, \ldots\}$$

Therefore, f is a nanogonal sum labeling and hence $B_{n,n}$ is a nanogonal sum graph.

Nanogonal labeling of $B_{11,11}$ is shown in Fig.6

Fig.6: Nanogonal labeling of $B_{11,11}$



3.6 Theorem

The graph $B_{n,n}$ is a Decagonal sum graph for $n \geq 2$.

Proof:

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Let $V\left(B_{n,n}\right)=\left\{u_i,\, 1\le i\le n\right\}\cup\left\{v_i,\, 1\le i\le n\right\}\cup\left\{u\right\}\cup\left\{v\right\}$

$E\left(B_{n,n}\right)=\left\{e_i,\, 1\le i\le 2n+1\right\}$ (See Figure 1)

First we label the vertices of $B_{n,n}$ as follows:

Define f : V ($B_{n,n}$) →N by

$$f(u)=0,$$
$$f(u_i)=i(4i-3),\, 1\le i\le n$$
$$f(v)=\left(n+1\right)\left(4n+1\right),$$
$$f(v_i)=\left(8n+9\right)i+4i(i-1),\, 1\le i\le n$$

Then the induced edge labels are,

$$f(e_i)=i(4i-3),\, 1\le i\le 2n+1,$$

Therefore,

$f^{+}\left(E\left(B_{n,n}\right)\right)=\left\{1,\, 10,\, 27,\, 52,\, 85,\, 126,\, 175,\, 232,\, 297,\, 370,\ldots\right\}$

Therefore, f is a decagonal sum labeling and hence $B_{n,n}$ is a decagonal sum graph.

Decagonal labeling of $B_{8,8}$ is shown in Fig.7

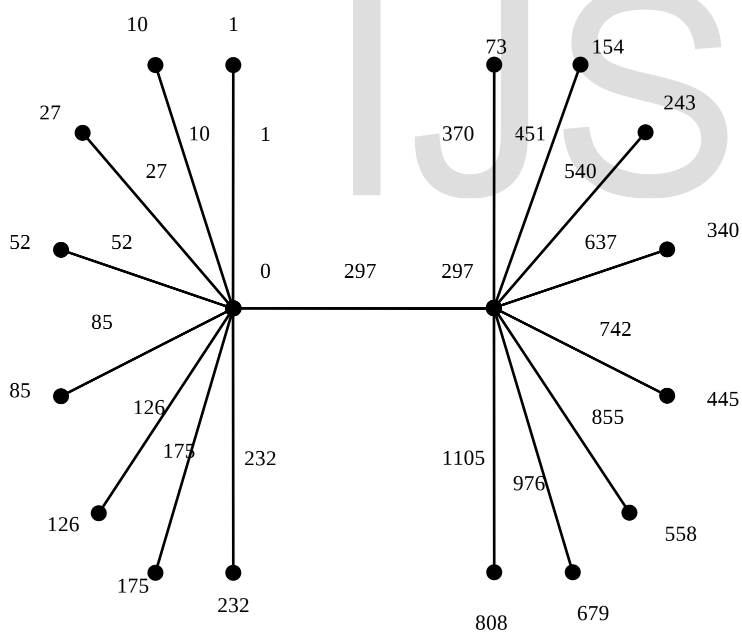

Fig.7: Decagonal labeling of $B_{8,8}$

Conclusion

The polygonal sum can be verified for many other graphs. Also some more polygonal sum labeling can be investigated.

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